

OFF-BOTTOM SOLIDS CONCENTRATION MEASUREMENTS: THE POSSIBLE IMPLICATIONS OF WILSON'S NEAR-WALL LIFT FORCE ON EROSIIVE PIPE WEAR

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Wear studies involving actual pipeline measurements are relatively few in number. One of the more comprehensive studies, conducted by Shook and co-workers, showed that the wear rate was directly proportional to the local particle flux (cv) and local wall normal force (F). Specifically, they showed that, for "coarse" particle ($d > 0.4$ mm) flows, the wear rates at the pipe invert were highest because this is the region where ($cv \cdot F$) is the greatest. Subsequent studies conducted by Wilson and co-workers on the "near-wall" lift force described the effects of pipe diameter, mixture velocity, particle diameter and concentration and carrier fluid viscosity on the magnitude of the lift force. The present study was conducted to investigate the variation of the coarse solids concentration at the bottom of the pipe as the mixture velocity was increased from a value near the deposition velocity ($V \sim V_c$) through the point where the near-wall lift force became important (V_{NWL}), and then to even higher velocities. The tests were conducted using a 75 mm pipeline loop and coarse particles of similar sizes but different densities and shapes (silica sand; aluminum oxide; zirconium silicate). The variation of the off-bottom solids concentration, measured using a collimated traversing density gauge, and pressure losses were also recorded. The analysis suggests that, for certain particle types, operation at velocities where the near-wall lift force is important may reduce wear at the pipe invert. Future tests are planned to further investigate this phenomenon.

KEY WORDS: Settling slurries, pipeline wear, particle concentration and velocity distributions, near-wall lift, stratification ratio

NOTATION

c	Local particle volume concentration (-)
cv	Local particle flux (m/s)
C_L	Particle lift coefficient (-)
C_r	Averaged in situ solids volume concentration (-)
C_{max}	Maximum (limiting) coarse particle volume fraction (-)
d	Particle diameter (m)
d_{50}	Mass median particle diameter of the coarse solids (m)
D	Pipe diameter (m)
f_w	Darcy friction factor for the carrier fluid (-)
F	Normal force (N)
g	Acceleration of gravity (m/s^2)

i_m	Slurry hydraulic gradient expressed in meters of water (m/m)
i_w	Hydraulic gradient for water (m/m)
k	Pipe wall roughness (m)
P	Pressure (Pa)
R	Stratification ratio, defined by Eq. (1) (-)
Re^*	Particle Reynolds number, defined by Eq. (4) (-)
S	Density ratio (ρ_s/ρ_f)
T	Temperature ($^{\circ}C$)
u^*	Particle shear velocity, defined by Eq. (5) (m/s)
v	Local particle velocity (m/s)
V	Average velocity (m/s)
V_c	Deposition velocity (m/s)
V_{NWL}	Cross-over velocity (m/s)
y	Vertical distance measured from pipe invert (m)
z	Axial coordinate (m)
μ	Viscosity (Pa.s)
θ	Dimensionless ratio, defined by Eq. (3) (-)
ρ	Density (kg/m^3)
Subscripts	
f	Carrier fluid
s	Solids

1. INTRODUCTION

In pipelines used to transport heterogeneous or settling slurries in Newtonian carrier fluids, methods to predict friction losses, deposition velocity and pump derating are reasonably well established (see, for example: Kaushal et al., 2005; ANSI/HI, 2005; Wilson et al., 2006; Spelay et al., 2013). However, despite their importance in industry, meaningful predictions of pipeline material loss through erosion/corrosion mechanisms are not possible (Cooke et al., 2000; Clark and Llewellyn, 2001; Lester et al., 2010). The Canadian oil sands industry alone loses more than $\$10^7$ annually to pipe replacement, preventative maintenance and lost production due to unexpected outages through wear-related equipment failure (Fuhr et al., 2014). (The magnitude of the estimate is such that it is immaterial whether it is expressed in US or Canadian currency equivalent.) There are numerous reasons why the magnitude of the problem has not led to improved wear models (Sadighian, 2015); certainly, one major issue is that many wear studies involve testing devices in which the hydrodynamics are not at all reflective of slurry pipeline flows. The aversion to conducting actual pipeline wear tests under controlled laboratory-scale conditions is because such studies are time-consuming, expensive and difficult to interpret since the particles degrade during testing (Shook et al., 1981; Cooke et al., 2000).

Despite the challenges associated with pipeline wear testing, some valuable results concerning the hydrodynamic aspects of slurry pipeline erosion are available in the literature. A relatively early but very important contribution was that of Shook et al.

(1990), who showed that for settling slurries that are associated with significant Coulombic (sliding bed) friction, the product of $[cv \cdot F]$ provides a good indication of wear rate. Clark (2002), in a seminal work, discussed the complexity of modeling erosion in suspension flows and clearly identified the main parameters having the greatest effect on slurry pipeline erosion. Lester et al. (2010) demonstrated how the effects of particle velocity and impact angle on erosion rate could be decoupled by studying the wear of a cylinder placed inside a slurry pipeline so that its base is mounted on the pipe wall and its side is normal to the direction of flow. These tests, along with complementary CFD simulations, were conducted using slurries containing 7% solids (by volume). This approach, although undoubtedly providing more information about the relative effects of particle velocity and impingement angle, is probably more difficult to interpret in a flow where the concentration is high and is asymmetric in the direction of gravity. Gnanavelu et al. (2011) described an innovative approach where they attempted to combine “geometry-independent” wear factors and CFD simulations to predict suspension wear. In this approach, relatively simple and efficient lab tests could be done using a jet impingement device to produce the wear factors. Unfortunately, this method fails to account for the increased wear observed at the pipe invert for contact load-dominated flows. More recently, Sadighian (2015) has shown how it may be possible to develop a relationship between erosive wear rate and the solids-related wall shear stress; in other words, the mechanisms that affect slurry friction loss should also dictate erosive wear.

If a connection between friction loss and wear can be made – even qualitatively – then the research conducted by Wilson and co-workers on the so-called near-wall lift force (e.g. Wilson et al., 2000; Wilson and Sellgren, 2003; Whitlock et al., 2004; Wilson et al., 2010) provides a compelling case study, particularly for flows where Coulombic friction is important. The onset of the near-wall lift force often can be seen in the decrease in solids concentration near the pipe wall, as shown in Figure 1 for results recently obtained at the Saskatchewan Research Council. Similar “turnarounds” in the concentration profile have been observed by many others, as described by Wilson et al. (2010). In that same paper, the authors developed a friction loss model based on the slurry stratification ratio, R :

$$R = \frac{i_m - i_w}{S - 1} \quad (1)$$

Their model is based on the reduction of the stratification ratio (R) using a particle lift coefficient:

$$R = \frac{1}{C_L \theta} = \frac{0.7}{\theta (\text{Re}^*)^{0.33}} \quad (2)$$

$$\theta = \frac{(3/32) f_w V^2}{g(S-1)d} \quad (3)$$

$$\text{Re}^* = \frac{\rho_f u^* d}{\mu_f} \quad (4)$$

$$u^* = \sqrt{(S-1)gd/6} \quad (5)$$

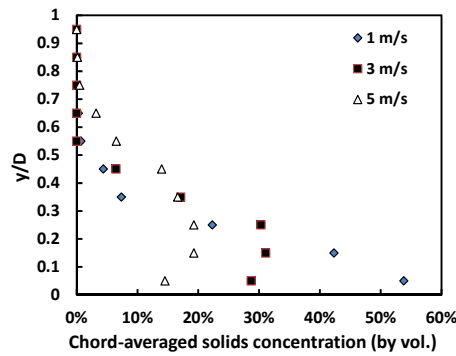


Fig.1 Concentration profiles for monosized Delrin spheres ($d = 3.34 \text{ mm}$; $\rho_s = 1400 \text{ kg/m}^3$) in water: $C_r = 9.7\%$; $D = 0.104 \text{ m}$; $T = 19.6^\circ\text{C}$ (unpublished SRC data)

The purpose of the present paper is to provide a preliminary evaluation of the potential effect that the near-wall lift force could have on the product ($cv \cdot F$) and therefore on wear rates in a slurry flow dominated by Coulombic (contact load) friction.

2. EXPERIMENTAL PROGRAM

Experiments were conducted using a horizontal 75.7 mm (ID) by 80 m (length) slurry pipeline loop / centrifugal slurry pump system located at the Saskatchewan Research Council's Pipe Flow Technology CentreTM. The details of the experimental setup can be found elsewhere (Sadighian, 2015). In this particular set of experiments, three key parameters were measured: slurry flow rate (magnetic flow meter), frictional pressure loss (differential pressure transducer) and chord-averaged solids concentration at $y/D = 0.05$ (traversing gamma ray densitometer). The length of the pressure drop test section was 3 m and it was located more than 100 pipe diameters from the nearest upstream flow disturbance. Three different water-based slurries were tested: silica sand, aluminum oxide and zirconium silicate. The particle properties are given in Table 1. The particle size distributions for the sand and aluminum oxide particles were obtained from sieve analysis and are shown in Figure 2. No size distribution is shown for the zirconium silicate (referred to as 'Si-Zi' in Table 1 and all subsequent figures showing data for that particular slurry) as the particles are essentially monosized and spherical. The in situ concentration of each slurry tested was held constant at 10%, a value chosen because the near-wall lift force is not attenuated under these conditions (Wilson et al., 2010).

The tests were conducted as follows: the desired mass of the chosen particle type was added to a water-filled loop (to provide the required in situ concentration) and then the loop was operated at a relatively high velocity to ensure the slurry concentration was uniform in the axial direction. The pump speed was then reduced slowly to determine the deposition velocity (V_c). The pump speed was then increased in step-wise increments. Each velocity condition was maintained for less than 4 minutes to reduce particle

degradation and consequent changes in carrier fluid properties. At each constant-velocity condition, the frictional pressure loss and chord-averaged solids concentration at $y/D = 0.05$ were measured. The pressure loss measurements are shown in Figure 3 and the same data are shown in the form of a stratification ratio plot in Figure 4. In the latter figure, the point at which the data cross the $R = i_w$ line indicates the operating conditions at which the lift force on a particle balances its submerged weight (Wilson et al., 2010). It also corresponds to the point at which a reversal in the concentration profile occurs (Wilson et al., 2010). The velocities at which $R = i_w$ for the three different slurries were 3.3 m/s (sand), 4 m/s (Al₂O₃) and 4.3 m/s (Si-Zi). In Fig. 5, the variation of the off-bottom solids concentration (at $y/D = 0.05$) is shown as a function of mixture velocity. It is interesting that the “crossover” velocity, V_{NWL} , corresponds almost exactly with a discontinuity in the slope dc/dV for the sand and Si-Zi particles. For the Al₂O₃ particles, the discontinuity can be observed as well, but occurs at a mixture velocity $V < V_{NWL}$. Clearly, the rather simplified approach that considers only particle immersed weight and particle lift (See Equations 1 to 4 and/or Wilson et al., 2010) cannot explain the behavior of the dc/dV curves. Since it is also not the main focus of the current paper, it must suffice to say that a more complete analysis of the vertical forces acting on a particle must be considered (e.g. Gillies and Shook, 1994; Spelay et al., 2015).

Table 1

Particle properties

Particle	d_{50} (mm)	ρ_s (kg/m ³)	C_{max}	Circularity	V_c (m/s)
Sand	0.420	2650	0.60	0.90	1.9
Al ₂ O ₃	0.425	3950	0.55	0.68	2.9
Si-Zi	0.450	3700	0.60	1	2.7

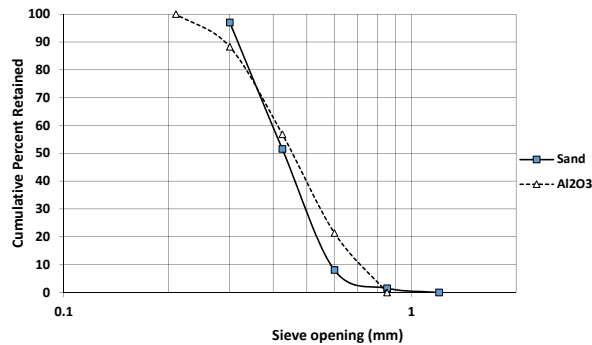


Fig.2 Measured particle size distributions for the Sil-4 sand and aluminum oxide particles tested in the present study. Not shown: Si-Zi, which are essentially monosized.

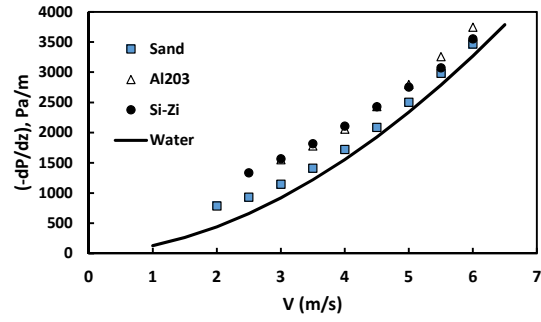


Fig.3 Measured pressure gradients for slurries flowing in horizontal test loop (symbols): $Cr = 10\%$, $D = 75.7 \text{ mm}$; $T = 20^\circ\text{C}$. Solid line shows pressure losses for water ($k = 7 \mu\text{m}$).

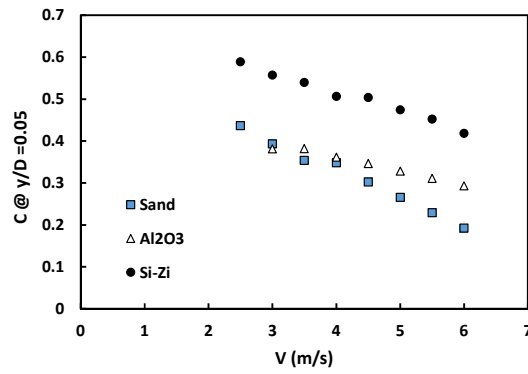


Fig.4 Stratification ratio plot for the slurries tested in the present study

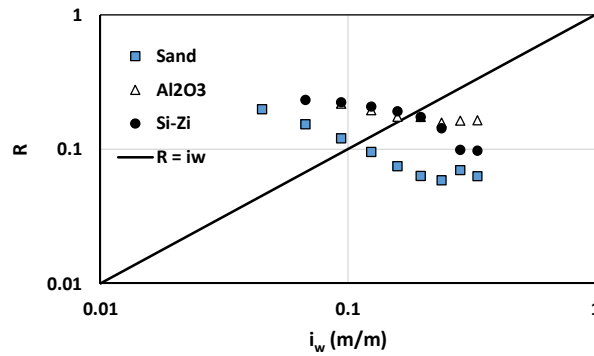


Fig.5 Chord-averaged solids concentration measured at a vertical position $y/D = 0.05$

3. ANALYSIS

3.1. PREDICTIONS OF THE NEAR-WALL MODEL

Friction loss predictions are not required to fulfill the main objective of this paper (which is presented in the next section); however, since the data are available, a brief analysis of the performance of the near-wall model developed by Wilson et al. (2010) is provided. For the conditions tested here, the slurry concentration is relatively low and the particles are larger than the viscous sublayer thickness but not so large as to provide a relative particle diameter (d/D) effect, the near-wall model of Wilson et al. (2010) is given by Equation (2), which was presented in the Introduction. The performance of the near-wall model is illustrated in the parity plot, shown here as Fig. 6. Generally, the model is better at extremes when the velocity is low (higher R values) or when the velocity is high and the near-wall lift effect is strong. The performance of the model observed here is similar to that described by Wilson et al. (2010), where the predicted stratification ratios in the intermediate velocity region were not as accurate as those of the low- and high-velocity conditions.

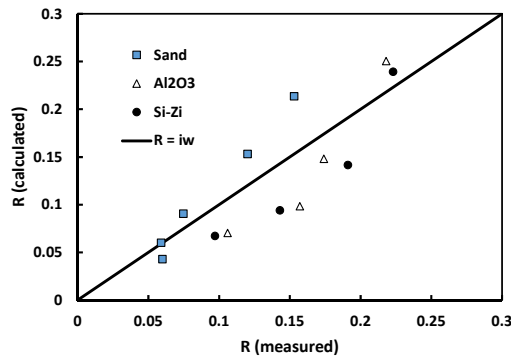


Fig.6 Comparison of the stratification ratios calculated from experimental measurements with predictions obtained using the near-wall model of Wilson et al. (2010).

3.2. POTENTIAL IMPLICATIONS TO EROSION PIPE WEAR

As described earlier, the study of Shook et al. (1990) clearly showed that the product of wall contact particle flux (cv) and normal force (F) is proportional to erosive wear rate for contact load-dominated slurry flows. In such cases, the wear in the bottom part of the pipe (say, 140 to 220° if 0° and 180° represent the top and bottom of the pipe, respectively) will be 2 to 2.5 times greater than anywhere else on the pipe circumference (Shook et al., 1990). In the following analysis, the change in the product [$cv \cdot F$] at the bottom of the pipe with increasing mixture velocity (V) is estimated. To do so, two simplifying assumptions are required: (i) the “near-wall” concentration is taken to be the value measured at $y/D = 0.05$ during the experiments; and (ii) the magnitude of the normal force (F) is proportional to the stratification ratio (Wilson et al., 2006), which is calculated from the pressure loss measurements (i.e. Fig. 3).

Clearly, the “near-wall” particle velocity is also required for this analysis. In theory, one could make such a measurement using a wall surface probe (Shook et al., 1990) or using other methods such as dual-plane Electrical Impedance Tomography (EIT) (Hashemi et al., 2014a). One could also calculate the value using CFD simulations, but such predictions (for velocity) are generally poor, especially at lower solids concentrations and mixture velocities (Hashemi et al., 2014b). In this particular analysis, however, a notable feature of contact load-dominated slurries is used to develop a relatively simple yet effective method of estimating velocity distributions. For coarse particle slurries that are of interest in the present study, it is well-known that V_c scales with $D^{0.5}$ (Gillies et al. 2000). The implication is that one should be able to compare velocity distributions in different pipe sizes, provided that C_r and V/V_c are roughly constant. A comparison such as this is shown in Figure 7, where scaled velocity measurements in three different pipelines are shown for conditions where $V/V_c \sim 1.3$ and $C_r \sim 20\%$. The convergence of the velocity measurements, particularly in the lower portion of the pipe, provide a method of estimating the “near-wall” velocity (taken at $y/D = 0.05$ to be consistent with the near-wall concentration assumption described earlier). Although only the scaled velocity distributions for $V/V_c \sim 1.3$ are shown here, a series of these curves was produced for a range of V/V_c values, which allowed for the prediction of the near wall velocity for all the velocity conditions tested during the present study.

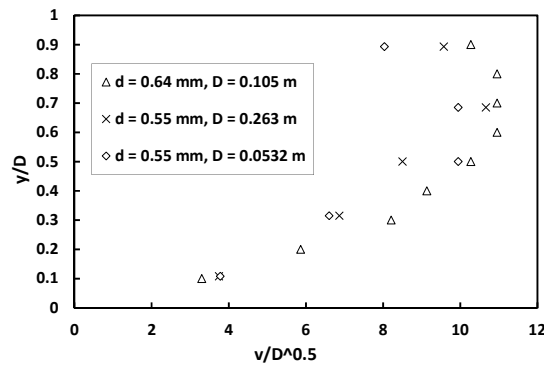


Fig.7 Scaled local velocities for slurries. 0.64 mm (0.105 m pipe): $C_r = 25\%$; $V/V_c = 1.25$; 0.55 mm (0.263 m pipe): $C_r = 25\%$; $V/V_c = 1.31$, 0.55 mm (0.0532 m pipe): $C_r = 15\%$; $V/V_c = 1.35$. Data taken from Gillies (1993).

Figure 8 demonstrates the quality of the predictions that can be made using the velocity scaling approach, using wall velocity measurements taken from Shook et al. (1990) for a slurry of 0.45 mm sand particles flowing in a 50 mm pipe at 2 m/s ($V/V_c = 1.15$) and an in situ solids concentration of 18%. For the comparison the centerline velocity predictions from the scaled velocity model were converted to near-wall velocities at the different circumferential positions using the “isovel” (isovelocity lines) approach outlined in Roco and Shook (1984). Considering the simplicity of the velocity scaling approach and the conversion from centerline velocity values to “near-wall” velocities, the agreement between the measurements and predictions is surprisingly good.

With the measured values of c ($y/D = 0.05$) and R , and the predictions of v from the scaled velocity maps, it is possible to show how the wear-indicating product $[cv \cdot R]$ varies as the mixture velocity (and impact of the near-wall lift) increases. In Figure 9, the product $[cv \cdot R]$ is normalized using the value calculated at an operating velocity that is “typical” for most coarse particle slurry pipelines operating in turbulent flow ($1.05 \leq V/V_c \leq 1.15$). The results show that the normalized wear-indicating product follows a clear trend: there is a slight increase (to about 1.05) at moderate values of V/V_c , and then a sharp decrease as V/V_c is increased beyond $V/V_c \sim 1.75$. The implication is that the wear in the bottom portion of the pipe for contact load slurries should be reduced since the product of (flux*normal force) is substantially reduced. It is interesting to note that if one applied the pseudo-homogeneous friction loss model developed by Talmon (2013), which states that the viscous sublayer is depleted of particles, one would obtain the high-velocity asymptote for Figure 9, i.e. a near-wall particle flux of zero. The advantage of the present approach is that one is able to see how the contact-load wear parameter reduces with increasing operating velocity.

At this point, some healthy skepticism regarding the above analysis is required, primarily for two reasons: (i) supporting experimental measurements have not yet been made; and (ii) any reduction in wear at the pipe invert may be completely overshadowed by the increased overall wear rates. The best possible implementation of higher-velocity operation may in fact be to strike a balance such that overall wear rates are slightly higher but are more uniform over the pipe circumference.

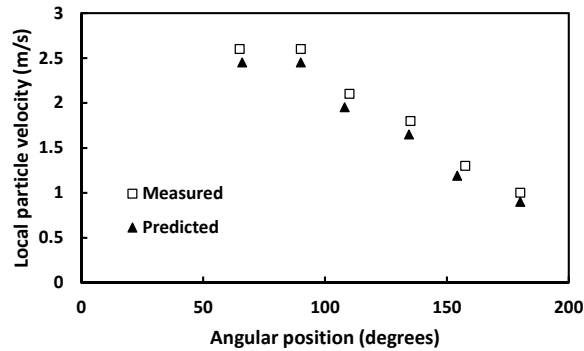


Fig.8 Comparison of the particle velocity measurements of Shook et al. (1990) with predictions obtained using the scaled velocity approach: $D = 53$ mm; $d = 0.45$ mm; $C_r = 18\%$; $V/V_c = 1.15$.

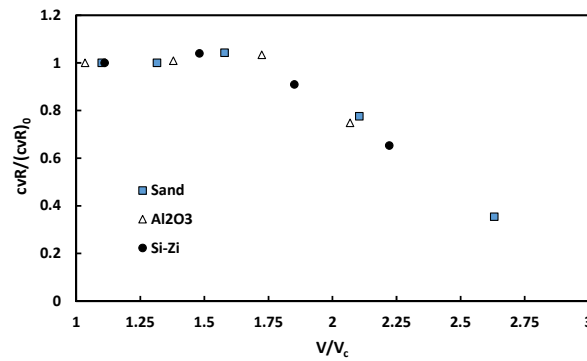


Fig.9 Variation of the contact-load wear parameter [$cv \cdot R$] normalized by the value obtained at "typical" operating velocities ($1.05 \leq V/V_c \leq 1.15$) for the three slurries tested during the present study.

4. CONCLUSIONS

In the present study, experiments were conducted using dilute, coarse particle slurries ($d \sim 0.425$ mm, $C_r = 10\%$) to evaluate the effect of high-velocity operation, and consequent onset of Wilson's near-wall lift, on the stratification ratio (R) and the chord-averaged concentration near the bottom of the pipe (i.e. at $y/D = 0.05$). Since no local particle velocities were measured, a simple method for predicting velocity distributions in contact load-dominated slurries was proposed. The relative product of particle flux and normal force [$cv \cdot R$] was calculated as a function of mixture velocity. The relative product decreases substantially at operating velocities $V/V_c > 1.75$. This finding may have important implications in terms of selecting operating conditions that reduce the asymmetric wear associated with coarse particle slurry flows, but most probably at the expense of increasing the overall wear rate. Complementary wear studies are presently underway.

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